Including Sustainability Criteria into the Multi-Supplier Newsvendor Problem: An MCDM and Bi-Objective Optimization Approach

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Abstract

We develop a single product single-period inventory control model with stochastic demand, in which a retailer buys, at the beginning of the single-period, a quantity of a perishable product from one or more than one supplier with limited capacity. The end customers' random demand that should be satisfied by the retailer is concentrated in a single period selling season during which every satisfied demand is charged a certain price by the retailer. At the end of the selling season, any remaining units are salvaged by the retailer at a salvage value and any unsatisfied demands incur a penalty shortage cost.

The problem is modeled using a bi-objective optimization framework. First, fuzzy TOPSIS is used in order to measure the closeness coefficients of all the available suppliers based on pre-determined sustainability criteria, that include green ones, such as the geographical distance between the production site of the supplier and the site of the retailer's warehouses, and social criteria such as the impact of the production activities on the local society. AHP is then used in order to determine the importance weights of the green and social penalty shortage and customer satisfaction coefficients. Furthermore, we model two independent multi-supplier newsvendor problems: a cost based sub-problem and a sustainability (green and social) based sub-problem. We solve each of these sub-problems analytically and we exhibit the structure of the optimal policy and therefore the optimal quantity to order from each supplier in both cases. We use then the comprehensive criterion method in order to solve the bi-objective model and we exhibit the structure of its optimal policy and the Pareto optimal solutions.

Furthermore, through a numerical study, we analyze the effect of some of the model parameters on the optimal policy and on the Pareto solutions. More particularly, we investigate the relative weight of the different criteria, the randomness of the demand, the difference in the costs between the supply options and the other economic parameters.

Keywords: : multi-supplier newsvendor, green and social sustainability, bi-objective optimization, MCDM, short life-cycle products.

1 Introduction

The newsvendor problem is a well studied single-period inventory control problem that aims at choosing the best ordering quantity of a single product usually with short life-cycle (Khouja, 1999). In a newsvendor problem, a retailer orders, before the beginning of a single period selling season, and usually from a single supplier, a quantity of a product to be used to satisfy

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the end customers' demand during the selling season. Each satisfied demand is charged a fixed price. Moreover, at the end of the selling season, each unsatisfied demand is lost and a corresponding penalty cost is incurred, and each unsold item is salvaged at a salvage price. Many extensions to the newsvendor problem exist in the related literature such as the ones with information updating (Cheaitou and Cheaytou, 2018), or with two ordering opportunities (Cheaitou et al., 2014). Another angle from which the newsvendor problem has been addressed is the consideration of a multiple suppliers. Recently, with the increase in the interest of the business organizations and the awareness of the society about the sustainability of the business activities, and more particularly the impact on the environment, researchers started introducing some environmental and social aspects in the inventory control models in general and in the newsvendor model in particular. Motivated by this increasing interest, we develop in this paper a multiple supplier framework for a newsvendor type problem, in which we consider, in addition to the economic performance, environmental and social aspects based on which the retailer can choose the best supplier(s) from which the orders can be places and their corresponding optimal quantities. To achieve this objective, we develop a bi-objective optimization framework in which the first objective aims to maximize the retailer's profit and the second the sustainability value of the purchased products. We characterize the complete structure of the optimal policy of the two single objective sub-problems and of the bi-objective problem using the Karush-Khun-Tucker conditions. Moreover, we implement the obtained optimal policy using Wolfram Mathematica and we obtain some managerial insights based on the numerical study. The reminder of this paper is structured as follows: Section 2 reviews the related literature. Section 3 provides the model description while Section 4 develops the model solution and analysis approach. Section 5 is dedicated to the numerical analysis of the proposed approach while Section 6 provides concluding remarks.

2 Literature review

The literature review can be classified mainly into two areas which are related to this work. The first category includes the works on Newsvendor models with multiple suppliers. The second category conducts a literature review on the environmental stainability in inventory problems.

In the real-life, many products, especially the goods such as books, candy, food, electronics are seasonal products (Mostard et al., 2005 ; Yao et al., 2005b) with a short life cycles. One of the classical single-period model in inventory control literature that fits this type of situation is the well-known newsvendor model. The single period-model consists of a retailer who orders a quantity of products from a supplier at the beginning of the selling season at a certain unit cost, which is delivered immediately in order to satisfy the uncertain end-customer demand. During the selling season, any satisfied demand is charged a unit price and any unsatisfied demand is lost and a lost sales cost is incurred. At the end of the selling season, any remaining units are salvaged at a salvage value which is less than the original price charged to the customers. The classical newsvendor model, where there is a relationship between a single/multiple retailer (or buyer) and single supplier, has obtained widespread attention in the literature (for exhaustive reviews on this topic, please refer to Porteus, 1990 ; Khouja, 1999; , Qin et al., 2011; and Choi, 2012;) and some extensions have been introduced to improve the model.

One of these extension is to include multiple suppliers in the newsvendor model. The first work dealing with multiple suppliers is that done by Agrawal and Nahmias (1997). They assumed that demand is deterministic, suppliers are unreliable and a fixed order cost is incurred for each supplier with a positive order. A group of works has also investigated the case of multiple suppliers including the scenario of unreliable suppliers (see, e.g. Chen et al., 2001; Babich et al., 2007; Yang et al., 2007;Dada et al., 2007; Burke et al., 2009; Merzifonluoglua and Fengb, 2014; van Delft and Vial, 2015; Park and Lee, 2016; Hu and Su, 2018). Minner (2003) offers a survey of research on multiple supplier inventory models in supply chain management.

Since one of the main characteristics of the model proposed in our paper is the newsvendor problem with multiple suppliers, we now focus on the previous works most closely related to our issue. In fact, Dada et al., (2007) study a newsvendor who procures from multiple suppliers, of whom some are unreliable. They show that the cost and the reliability impact the optimal ordering quantities in different manners: no order will be made by the retailer when he faces suppliers with excessive costs, and this is no matter the reliability level. On the contrary, some order will be made by the retailer when he faces suppliers with low costs, but the size of the order depends on their reliability. Burke et al., (2009) have pointed out the same result. Merzifonluoglu and Feng, (2014) show that cost might not be the order qualifier if there is a fixed ordering cost.

Another extension to the newsvendor model belongs to the category of green or sustainable inventory in which environmental and social aspects are added to the traditional inventory problems. Arikan and Jammernegg (2014) consider the single period inventory model with product carbon foot print constraint. They specify an upper bound for the carbon foot print as a benchmark derived either from the company's environmental target or from an industry standard. Sel et al., (2017) study the planning and scheduling of food production-distribution operations with environmental and social concerns in addition to economical ones. Tsao et al. (2017) study the newsvendor models that take into account carbon emissions, trade credit, and product recycling, in which the credit period affects the risk of default with uncertain demand. They determine the optimal ordering quantity, credit period, and recycle price in order to maximize total profits. Konur et al. (2017) analyze an integrated inventory management and delivery scheduling problem in a stochastic demand environment with economic and environmental considerations. They use a bi-objective continuous review inventory control model with order splitting among multiple suppliers using two different delivery scheduling policies: sequential splitting and sequential delivery. Ma et al., (2018) study an issue of dynamic procurement planning under the carbon tax in a supply chain. A manufacturer needs to select appropriate suppliers to satisfy the random demand. The authors study therefore two problems: how to make the optimal decision on order quantity and how to select appropriate suppliers for a manufacturer, in consideration of a carbon tax.

Other papers addressed the sustainable or the green inventory or supply problem with different objectives such as Mafakheri et al. (2011), Gavronski et al. (2011), MirzapourAl-e-hashem and Rekik (2014), Seroka-Stolka (2016), Garcia-Alvarado et al. (2017), and Tang et al. (2018).

To the best of our knowledge, none of the papers discussed above has considered a multiple supplier newsvendor framework with consideration of the sustainability aspects and a complete characterization of the optimal policy which this paper does.

3 Model description

We consider a single product single-period bi-objective model for identifying the best ordering policy for a retailer (or decision maker or vendor) served by multiple suppliers subject to individual production capacities and faced with a stochastic demand. The model aims to determine the best suppliers to buy from and the amount that should be ordered from each selected supplier, taking into account two criteria: the cost criteria of each supplier and the sustainability criteria including green and social criteria. Thus, the model consists of three stages: the single-objective cost problem, the single-objective sustainability problem and the bi-objective optimization problem.

Suppose there are n suppliers from which the retailer can source. For the retailer, let p be the unit selling price of an item, s the unit salvage value for any remaining items at the end of selling season, and π the unit penalty shortage cost for every unsatisfied demand. Let c_i be the cost owed to supplier i and K_i the production capacity of each supplier i. Finally, the probability density function (PDF) and the cumulative density function (CDF) of the demand D are denoted by $f(D)$ and $F(D)$ respectively.

First, in the single-objective cost problem, the retailer aims to maximize its expected profit by choosing the vector of nonnegative order quantities $\mathbf{Q} = (Q_1, Q_2, ..., Q_n)$ which constitutes the decision variable of the vendor.

Second, the single-objective sustainability problem allows the retailer to select his suppliers based on the supplier's green and social performance. To this end, we use fuzzy TOPSIS (introduced by Hwang and Yoon (1981)) to assign preference weights w_i based on green and social criteria for each supplier i $(i = 1, ..., n)$. The assignment of the weights is based on the available knowledge and expertise of the decision makers as well as the relative importance of each criterion to the company. Decision makers can assign weights to the suppliers with respect to the criteria using available historical data, the capability studies on the suppliers, and laboratory testing and analysis of the product to be purchased (see CheaitouHamdan CIE, COR).

In addition, the retailer uses AHP to assign an importance total weight (w^{GS}) to the two sets of criteria, namely, the set of green criteria and the set of social criteria. This total weight includes the importance given by the company to selecting green or sustainable suppliers and transporters. It also covers the importance to the company of the environmental impact of disposal or salvage of the unsold products. It may result from the negative impact of unsold product on the use of resources used in the manufacturing of the product or because of the additional transportation of unsold products to a parallel market. Moreover, the decision maker uses AHP to determine two another importance weights: the first importance weight is given by the company to the negative impact of a shortage (π^c) that may represent the perception of decision makers of the negative impact on the companies image of shortage. The second importance weight w^c is given by the company to the positive impact of a satisfied customer on the company's image.

We define w_i^{GS} also as the performance level (preference weight) of supplier i based on the green and social criteria. It reflects the importance weight of the green and social aspects to the company (w^{GS}) as well as the relative performance of supplier i in the green and social aspects compared to the other suppliers (w_i). Finally, w_i^{GS} can be expressed as

$$w_i^{GS} = w^{GS} \times w_i, \tag{1}$$

where w_i is obtained from fuzzy TOPSIS based on the green and social criteria as mentioned previously.

Remark 1 *Note that π^c and w^c may have the same value. Moreover, it is worth noting that w^{GS},*

w^c and π^c are determined by pairwise comparison which is why we select AHP. This allows the decision maker to gauge (compare) the green and social aspect of the inventory systems management (w^{GS}) with the customer satisfaction aspect (w^c, π^c) that reflects an economic performance.

A summary of the notations used in our model are listed in Table 1.

Table 1: **Summary of Notation.**

n	Total number of suppliers.
c_i	Cost per unit owed to supplier i.
p	Selling price per unit charged by the buyer.
s	Salvage value.
π	Penalty cost of loss goodwill.
D	Stochastic demand.
$f(.)$	Probability density function (PDF) for demand.
$F(.)$	Cumulative probability distribution function (CDF) for demand.
Q_i	Quantity ordered from supplier i.
$\sum_{i=1}^{n} Q_i$	Total quantity ordered among the suppliers.
$\mathbf{Q} \equiv \{Q_1, \ldots, Q_n\}$	Vector of quantities ordered among the suppliers.
K_i	Production capacity of supplier i.
w^{GS}	Total importance weight of the green and social aspects obtained from AHP.
π^c	Importance weight given by the company to the negative impact of a shortage.
w^c	Importance weight given by the company to the positive impact of a satisfied customer on the company's image.
w_i	Relative performance of supplier i based on the green and social criteria from fuzzy TOPSIS.
w_i^{GS}	Performance level of supplier i based on the green and social criteria. It can be expressed as: $w_i^{GS} = w^{GS} \times w_i$.

4 Model analysis

4.1 Cost problem

In this section, we present the first stage represented by the cost problem which is equivalent to the retailer's problem taking into account the fact that he faces n suppliers with a capacity production K_i for each supplier i. First, we show that the expected profit function is jointly concave. Then, we analytically solve the retailer's optimal decision problem and characterize its solution. Subsequently, we determine the optimal quantities that the retailer should order from supplier i (whether that be zero or not) and thus we derive the optimal ordering policy for all the possible scenario that can be faced by the retailer.

Let us first denote by Q_i^* the optimal value of Q_i and let us denote by $\mathbf{Q}^*$=$\{Q_i^*\}$ the corresponding vector of decision variables. As it is known in the newsvendor problem, the retailer has to make decisions before the beginning of the selling season which is the case in this cost model. In addition, the retailer has the possibility to order among n independent suppliers. His first objective is to maximize the expected profit for the selling season denoted $\Pi(\mathbf{Q})$ where

$$\begin{aligned}\Pi(\mathbf{Q}) = \quad & p \quad \int_0^{\sum_{i=1}^n Q_i} D f(D) dD + p \sum_{i=1}^n Q_i \int_{\sum_{i=1}^n Q_i}^{\infty} f(D) dD \\ + \quad & s \int_0^{\sum_{i=1}^n Q_i} \left(\sum_{i=1}^n Q_i - D \right) f(D) dD - \sum_{i=1}^n c_i Q_i \\ - \quad & \pi \int_{\sum_{i=1}^n Q_i}^{\infty} \left(D - \sum_{i=1}^n Q_i \right) f(D) dD.\end{aligned} \tag{2}$$

The retailer's optimization problem is then defined as

$$\max \qquad \Pi(\mathbf{Q}) \tag{3}$$

$$\text{subject to} \quad 0 \leq Q_i \leq K_i \quad \text{for all i.} \tag{4}$$

Taking the derivative of (2) with respect to Q_i $(i = 1, .., n)$, it yields

$$\frac{\partial \Pi(\mathbf{Q})}{\partial Q_i} = (p + \pi - c_i) - (p + \pi - s) F \left(\sum_{i=1}^n Q_i \right), \tag{5}$$

Setting the partial derivative (5) equal to zero, we obtain the threshold level associated with supplier i where F^{-1} is the inverse cumulative distribution function (CDF) of demand:

$$Y_i = F^{-1} \left(\frac{p + \pi - c_i}{p + \pi - s} \right) \tag{6}$$

The question now is how much the retailer has to order from supplier i $(i = 1, ..., n)$ (whether that be zero or otherwise). To this end, we index the suppliers from the most to least expensive so that

$$c_{(n)} > ... > c_{(i)} > ... > c_{(1)}, \tag{7}$$

where c_i, denotes the unit order cost per received product from supplier i. The retailer chooses then a per-unit selling price p with $p > c_{(n)}$. If the realized demand is less than the retailer's available stock, then the retailer has the option to return the leftovers at a per-unit salvage value s with $s < c_{(1)}$.

By definition of Y_i, the indexing (7) implies that

$$Y_{(n)} < ... < Y_{(i)} < ... < Y_{(1)}.$$

We prove in Proposition 1 that the expected objective function of the cost model is jointly concave.

Proposition 1 *The expected objective function $\Pi(\boldsymbol{Q})$ defined in (2) is a jointly concave function with respect to Q_i, $i = 1, ..., n$.*

Proof 1 *The Hessian matrix of $\Pi(\boldsymbol{Q})$ with respect to Q_i ($i = 1, ..., n$) is the $n \times n$ matrix given by*

$$H_{\Pi}(\boldsymbol{Q}) = -(p + \pi - s) f(\sum_{i=1}^{n} Q_i) J_n \tag{8}$$

where J_n is $(n \times n)$ matrix of ones.

For each vector

$$\boldsymbol{V} = (V_1, ..., V_n) \subset \mathbb{R}^n,$$

we find

$$\boldsymbol{V}^T \left(H_{\Pi}(\boldsymbol{Q}) \right) \boldsymbol{V} = -(p + \pi - s) f(\sum_{i=1}^{n} Q_i)(\sum_{i=1}^{n} V_i)^2.$$

Since $s < \pi$ which means that $(p + \pi - s) > 0$, we have

$$\boldsymbol{V}^T \left(H_{\Pi}(\boldsymbol{Q}) \right) \boldsymbol{V} \leq 0.$$

We conclude that the matrix $H_{\Pi}(\boldsymbol{Q})$ is semi-definite negative. Consequently, the objective function $\Pi(\boldsymbol{Q})$ is jointly concave with respect to Q_i ($i = 1, ..., n$), which completes the proof. □

4.1.1 Karush-Khun-Tucker conditions

In this part, we present the Karush-Khun-Tucker (KKT) conditions corresponding to the cost problem. We will resort to these conditions to derive the optimal ordering policy.

Without loss of generality, we use in the equations below the index j instead of i in order not to have any ambiguity in the proofs presented later.

Since we deal with a special case where there are positivity constraints of the form

$$Q_j \geq 0 \Leftrightarrow -Q_j \leq 0,$$

it is common to treat nonnegativity implicitly; thus, we ignore the positivity constraints and instead of using the "classical" Lagrangian, we use the "modified" Lagrangian associated with problem (2) and defined as

$$L(Q_j,\lambda_j) = \Pi(\mathbf{Q}) - \sum_{j=1}^{n} \lambda_j (Q_j - K_j) \text{ where } \mathbf{Q} \equiv \{Q_1, ..., Q_n\}. \tag{9}$$

Note that Q_j , $j = 1,...,n$ are the variables and λ_j are the Lagrange multipliers.
Any optimal order quantity Q_j, for every $j = 1,...,n$ must satisfy the following

$$\frac{\partial L}{\partial Q_j} = \frac{\partial \Pi}{\partial Q_j} - \lambda_j \leq 0 \tag{10}$$

$$Q_j \frac{\partial L}{\partial Q_j} = Q_j \left(\frac{\partial \Pi}{\partial Q_j} - \lambda_j \right) = 0 \tag{11}$$

$$\frac{\partial L}{\partial \lambda_j} \geq 0 \Leftrightarrow Q_j \leq K_j \tag{12}$$

$$\lambda_j \frac{\partial L}{\partial \lambda_j} = \lambda_j (Q_j - K_j) = 0 \tag{13}$$

$$\lambda_j \geq 0. \tag{14}$$

Remark 2 (Sufficiency of KKT conditions) *It is shown in Proposition 1 that the function* $\Pi(Q)$ *is jointly concave with respect to* Q_j*,* $j = 1,...n$*. In addition, it is obvious that the constraints are convex in* Q_j *since they are given by linear functions. Thus the KKT conditions are necessary and sufficient conditions (Bazaraa et al, 1993) of maximization of* $\Pi(\boldsymbol{Q})$*.*

4.1.2 Structure of the optimal policy

In this part, we provide some properties of optimal selection and we characterize the optimal ordering policy. We also derive explicit expressions for the optimal purchase quantity by three different scenarios that can faced the retailer.

The following result provides an interesting property of optimal selection.

Proposition 2 *Consider the model with* n *suppliers. Assuming that there are two consecutive suppliers with respective costs indexed* $c_{(i)}$ *and* $c_{(i+1)}$ *such that* $c_{(i+1)} > c_{(i)}$*. If* $Q^*_{(i)} = 0$*, then* $Q^*_{(i+1)} = 0$ *for* $i = 1,...,n-1$*.*

Proof 2 *Assuming that* $Q^*_{(i)} = 0$*. Then, according to (KKT) conditions: Eqs: (10) and (11), it is inferred that* $\frac{\partial \Pi}{\partial Q^*_{(i)}} - \lambda_{(i)} < 0$*. In addition, from condition (14), we have necessarily* $\frac{\partial \Pi}{\partial Q^*_{(i)}} < 0$*. This implies that*

$$Y_{(i)} < \sum_{i=1}^{n} Q_i. \tag{15}$$

Now, consider

$$\frac{\partial \Pi}{\partial Q^*_{(i+1)}} = (p+\pi-c_{(i+1)}) - (p+\pi-s)F\left(\sum_{i=1}^{n} Q_i\right).$$

Since $c_{(i+1)} > c_{(i)} \Rightarrow Y_{(i+1)} < Y_{(i)}$*. Thus, from (15),* $Y_{(i+1)} < \sum_{i=1}^{n} Q_i$*, which means that* $(p+\pi-c_{(i+1)}) - (p+\pi-s)F\left(\sum_{i=1}^{n} Q_i\right) < 0 \Rightarrow \frac{\partial \Pi}{\partial Q_{(i+1)}} < 0$*. It yields according to KKT conditions that*

$$\frac{\partial \Pi}{\partial Q_{(i+1)}} - \lambda_{(i+1)} < 0.$$

Finally, thanks to (11), we deduce that $Q^*_{(i+1)} = 0$*.* □

Proposition 2 gives an indication on the way the retailer should follow to select suppliers. In fact, based on the indexing of the suppliers, it is optimal for the retailer to choose firstly the least expensive supplier and to add gradually to its selection set another suppliers, one by one according to how cheapest each supplier is. Moreover, according to Proposition 2, we can say also that when it is optimal for the retailer not to place an order with a supplier offering a cost indexed $c_{(i+1)}$, it will be also optimal for him not to place an order with any other supplier offering a cost greater than $c_{(i+1)}$. To generalize this point, we can express it mathematically as follows.

$$\text{If} \quad Q^*_{(i+1)} = 0 \quad then \quad Q^*_j = 0 \quad \text{for} \quad \text{all} \quad j > i+1. \tag{16}$$

4.1.2.1 First scenario: $Y_{(i+1)} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$ for $i = 1, ..., n-1$

We establish first the following property in Lemma 1.

Lemma 1 *Consider the model with* n *suppliers. If supplier i is given such that* $Y_{(i+1)} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$ *for* $i = 1, ..., n-1$*, we have*

$$\sum_{j=1}^{n} Q_{(j)} = \sum_{j=1}^{i} K_{(j)}.$$

Proof 3 *The proof is made by induction. Let us first make the proof for a particular supplier indexed* $i = 3$ *with a capacity production* $K_{(3)}$ *such that*

$$Y_{(4)} < K_{(1)} + K_{(2)} + K_{(3)} < Y_{(3)}. \tag{17}$$

From the indexing of suppliers, we deduce that (17) implies

$$K_{(1)} + K_{(2)} < Y_{(2)} \quad \textit{and} \quad K_{(1)} < Y_{(1)}$$

From the concavity of $\Pi(Q)$*, we can see that the quantities should be ordered are necessarily* $Q_{(1)} = K_{(1)}$*,* $Q_{(2)} = K_{(2)}$*,* $Q_{(3)} = K_{(3)}$ *and* $Q_{(4)} = 0$ *and this is regardless of* $K_{(4)}$ *and so on. Hence, we find that* $\sum_{j=1}^{n} Q_{(j)} = \sum_{j=1}^{3} K_{(j)}$*.*

Assume that the result holds for i $= k$, i.e, $\sum_{j=1}^{n} Q_{(j)} = \sum_{j=1}^{k} K_{(j)}$. Then, it is easy to show for the same reasons mentioned previously that for i $= k+1$, it yields
$Q_{(1)} = K_{(1)}$,...,$Q_{(k)} = K_{(k)}$, $Q_{(k+1)} = K_{(k+1)}$ *and* $Q_{(k+2)} = 0$ *and this is regardless of* $K_{(k+2)}$ *and so on. By the principle of mathematical induction, the proof is completed.* □

Proposition 3 *Consider the model with n suppliers. If supplier i is given such that* $Y_{(i+1)} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$ *for* $i = 1, ..., n-1$, *then* $Q^*_{(j)} = 0 \; \forall \; j > i$.

Proof 4 *The proof is carried out by contradiction. Suppose that for a particular j with $j > i$, there exists $Q^*_{(j)} > 0$. Based on Proposition 2, it is sufficient to establish the proof for $j = i+1 > i$. Thus, based on (KKT) conditions, if $Q^*_{(i+1)} > 0$, then, due to the complementary slackness condition (11), inequality (10) must be fulfilled as the equality*

$$\frac{\partial \Pi}{\partial Q^*_{(i+1)}} - \lambda_{(i+1)} = 0.$$

On one hand, we have using (5)

$$\frac{\partial \Pi(\boldsymbol{Q})}{\partial Q^*_{(i+1)}} = (p + \pi - c_{(i+1)} - (p + \pi - s)F\left(\sum_{j=1}^{n} Q_j\right) \tag{18}$$

On the other hand, we find from the monotonicity of $F(.)$

$$\sum_{j=1}^{i} K_{(j)} > Y_{(i+1)} \Rightarrow F(\sum_{j=1}^{i} K_{(j)}) > F(Y_{(i+1)}) \tag{19}$$

Thus, it yields using (6)

$$(p + \pi - c_{(i+1)}) - (p + \pi - s)F(\sum_{j=1}^{i} K_j) < 0. \tag{20}$$

But, from Lemma 1 and using (20), we obtain the following inequality

$$\frac{\partial \Pi(\boldsymbol{Q})}{\partial Q^*_{(i+1)}} < 0 \tag{21}$$

It is obvious that (21) contradicts condition (14). Consequently, Proposition 3 is proved. □

To interpret 3, we note first that the condition $Y_{(i+1)} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$ is equivalent to

$$(p + \pi - c_{(i+1)}) < (p + \pi - s)F(\sum_{j=1}^{i} K_j) < (p + \pi - c_{(i)}). \tag{22}$$

where $(p + \pi - c_{(i)})$ and $(p + \pi - c_{(i+1)})$ are the underage costs when ordering from two consecutive suppliers with their cost indexed respectively $c_(i)$ and $c_(i+1)$ with $c_{(i)} > c_{(i+1)}$. In addition, $(p+\pi-s)F(\sum_{j=1}^{i} K_j)$ represents the overage cost associated with an ordering starting from the least expensive supplier until supplier i

Proposition 4 *Consider the model with n suppliers. If supplier i is given such that $Y_{(i+1)} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$ for $i = 1,...,n-1$, then $Q^*_{(j)} = K_{(j)}\ \forall\ j \leq i$.*

Proof 5 *The proof is made by induction. For a given supplier $i = 1$ such that $Y_{(2)} < K_{(1)} < Y_{(1)}$, it is easy to show that $Q^*_{(1)} = K_{(1)}$ (we omit the details for sake of brevity). Let us show the proof for $i = 2$ such that*

$$Y_{(3)} < K_{(1)} + K_{(2)} < Y_{(2)}. \tag{23}$$

Applying (KKT) conditions with a cost and a production capacity indexed respectively $c_{(2)}$ and $K_{(2)}$ yield

$$\frac{\partial \Pi}{\partial Q^*_{(2)}} - \lambda_{(2)} \leq 0 \tag{24}$$

$$Q^*_{(2)}\left(\frac{\partial \Pi}{\partial Q^*_{(2)}} - \lambda_{(2)}\right) = 0 \tag{25}$$

$$Q^*_{(2)} \leq K_{(2)} \tag{26}$$

$$\lambda_{(2)}(Q^*_{(2)} - K_{(2)}) = 0 \tag{27}$$

$$\lambda_{(2)} \geq 0. \tag{28}$$

From (27), either $Q^*_{(2)} - K_{(2)} = 0$ or $\lambda_{(2)} = 0$.

When $Q^*_{(2)} = K_{(2)}$ and $\lambda_{(2)} \neq 0$, Eqs. (24) and (25) yield $\frac{\partial \Pi}{\partial Q^*_{(2)}} = \lambda_{(2)}$. It remains to show that $\lambda_{(2)} > 0$. From the monotonicity of $F(.)$, we get

$$Y_{(3)} < K_{(1)} + K_{(2)} < Y_{(2)} \Rightarrow F(Y_{(3)}) < F(K_{(1)} + K_{(2)}) < F(Y_{(2)}) \tag{29}$$

Thus, it yields using (6)

$$\frac{p+\pi-c_{(3)}}{p+\pi-s} < F(K_{(1)} + K_{(2)}) < \frac{p+\pi-c_{(2)}}{p+\pi-s}.$$

Hence,

$$(p+\pi-s)F(K_{(1)} + K_{(2)}) - (p+\pi-c_{(2)}) < 0 \tag{30}$$

According to Lemma 1, the partial derivative with respect to $Q^*_{(2)}$ is equal

$$\begin{aligned} \frac{\partial \Pi(\boldsymbol{Q})}{\partial Q^*_{(2)}} &= (p+\pi-c_{(2)}) - (p+\pi-s)F(\sum_{j=1}^{n} K_{(j)}) \\ &= (p+\pi-c_{(2)}) - (p+\pi-s)F(\sum_{j=1}^{2} K_{(j)}) > 0 \quad (thanks \quad to \quad (30)) \end{aligned} \tag{31}$$

which is in agreement with condition (28). Hence, all the (KKT) conditions are fulfilled. Since the objective function Π is jointly concave with respect to Q_i $(i = 1,...,n)$ and the constraints are linear, the solution $Q^*_{(2)} = K_{(2)}$ is an optimal solution.

In addition, we have by (23) that $K_{(1)} + K_{(2)} < Y_{(2)} < Y_{(1)}$ which implies, given the indexing of suppliers, that $K_{(1)} < Y_{(1)}$. Thus, we return to the case $i = 1$ and following the same arguments presented previously, we get $Q^*_{(1)} = K_{(1)}$ is also an optimal solution. Therefore, we have $Q^*_{(1)} = K_{(1)}$, $Q^*_{(2)} = K_{(2)}$ $\forall$ $j \leq 2$.

By the principle of mathematical induction, the proof is completed. □

4.1.2.2 Second scenario: $\sum_{j=1}^{i} K_{(j)} < Y_{(i+1)}$ for $i = 1,...,n-1$

Corollary 1 *Consider the model with n suppliers. If supplier i is given such that $\sum_{j=1}^{i} K_{(j)} < Y_{(i+1)}$ for $i = 1,...,n-1$, then $Q^*_{(j)} = K_{(j)}$ $\forall$ $j \leq i$.*

Proof 6 *The proof of this corollary follows directly from Proposition 4. In fact, according to the indexing of suppliers, $\sum_{j=1}^{i} K_{(j)} < Y_{(i+1)} \Rightarrow \sum_{j=1}^{i} K_{(j)} < Y_{(i)}$. Hence, the proof is carried out exactly as in Corollary 4.* □

Lemma 2 *Consider the model with n suppliers. If supplier i is given such that $\sum_{j=1}^{i} K_{(j)} < Y_{(i+1)}$ for $i = 1,...,n-1$, then $Q^*_{(j)} > 0$ $\forall$ $j > i$.*

Proof 7 *The proof is carried out by contradiction. It suffices to show that there is a contradiction for a particular $j > i$. Thus, suppose there exists $j = i+1 > i$ such that $Q^*_{(i+1)} = 0$. From Proposition 2, it yields that $Q^*_{(j)} = 0$ for all $j > i+1$. Hence, thanks to Corollary 1, it yields*

$$\sum_{j=1}^{n} Q^*_{(j)} = \sum_{j=1}^{i} Q^*_{(j)} + \sum_{j=i+1}^{n} Q^*_{(j)} = \sum_{j=1}^{i} K_{(j)}.$$

On one hand, (KKT) conditions (10) and (11) give that

$$\frac{\partial \Pi}{\partial Q^*_{(i+1)}} - \lambda_{(i+1)} < 0. \tag{32}$$

On the other hand, given that we assume $Q^*_{(i+1)} = 0$, (KKT) optimality condition (13) for $Q_{(i+1)}$ implies that

$$-\lambda_{(i+1)} K_{(i+1)} = 0 \Rightarrow \lambda_{(i+1)} = 0 \quad \textit{since} \quad K_{(i+1)} \neq 0 \tag{33}$$

Therefore, condition (32) becomes

$$\frac{\partial \Pi}{\partial Q^*_{(i+1)}} < 0. \tag{34}$$

However, due to the condition $\sum_{j=1}^{i} K_{(j)} < Y_{(i+1)}$ for $i = 1,...,n-1$, we obtain from the monotonicity of $F(.)$

$$F(\sum_{j=1}^{i} K_{(j)}) < F(Y_{(i+1)}) \Rightarrow F(\sum_{j=1}^{i} K_{(j)}) < \frac{p + \pi - c_{(i+1)}}{p + \pi - s}$$

Therefore, $(p+\pi-s)F(\sum_{j=1}^{i} K_{(j)}) - (p+\pi-c_{(i+1)}) < 0 \Rightarrow \dfrac{\partial \Pi}{\partial Q^*_{(i+1)}} > 0$*, which contradicts (34). Thus, Proposition 2 is proved.* □

4.1.2.3 Third scenario: $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ for $i = 1,...,n$

Proposition 5 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1,...,n$*, then* $Q^*_j = 0 \ \forall \ j > i$*.*

Proof 8 *This result follows directly from Proposition 3. In effect, since* $\sum_{j=1}^{i} K_{(j)} > Y_{(i)}$*, it can be derived according to the indexing of suppliers, that* $\sum_{j=1}^{i} K_{(j)} > Y_{(i+1)}$*. Thus, we fall in the same case presented in Proposition 3 and similar proof will follow.* □

Proposition 6 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1,...,n$*, then* $Q^*_j = K_{(j)} \ \forall \ j < i$*.*

Proof 9 *Since* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)}$*, we deduce by the indexing of suppliers that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i-1)}$*. Hence, it can be shown by induction similarly to Proposition 4 that* $Q^*_j = K_{(j)} \ \forall \ j \leq i-1$ *and it is omitted for brevity.* □

Proposition 7 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1,...,n$*, then* $Q^*_{(i)} = Y_{(i)} - \sum_{j=1}^{i-1} K_{(j)}$*.*

Proof 10 *Let us first write the (KKT) conditions for the supplier i with a cost and capacity production indexed respectively as* $c_{(i)}$ *and* $K_{(i)}$*:*

$$\frac{\partial \Pi}{\partial Q_{(i)}} - \lambda_{(i)} \leq 0 \tag{35}$$

$$Q_{(i)} \left(\frac{\partial \Pi}{\partial Q_{(i)}} - \lambda_{(i)} \right) = 0 \tag{36}$$

$$Q_{(i)} \leq K_{(i)} \tag{37}$$

$$\lambda_{(i)}(Q_{(i)} - K_{(i)}) = 0 \tag{38}$$

$$\lambda_{(i)} \geq 0. \tag{39}$$

From (38), there are two cases to check:
<u>Case 1:</u> $\lambda_{(i)} = 0$ *and* $Q_{(i)} \neq K_{(i)}$ *then, condition (36) is reduced to*

$$Q_{(i)} \left(\frac{\partial \Pi}{\partial Q_{(i)}} \right) = 0 \tag{40}$$

Thus, we have two possibilities:

First possibility: Assume $Q_{(i)} = 0$. *In this case,* $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should be negative (according to condition (35)). But, we have*

$$\begin{aligned}\frac{\partial \Pi}{\partial Q_{(i)}} &= (p+\pi-c_{(i)})-(p+\pi-s)F(\sum_{j=1}^{n} Q_j) \\ &= (p+\pi-c_{(i)})-(p+\pi-s)F(\sum_{j=1}^{(i-1)} K_j) \; (\textit{thanks to Proposition 5 and Proposition 6}) \\ &> 0 \quad (\textit{since} \sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} \textit{ and using the monotonicity of } F(.)) \end{aligned} \tag{41}$$

which violates condition (35).

Second possibility: Assume $Q_{(i)} \neq 0$. *In this case,* $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should be equal zero. Hence,*

$$\begin{aligned}\frac{\partial \Pi}{\partial Q_{(i)}} &= (p+\pi-c_{(i)})-(p+\pi-s)F\left(\sum_{j=1}^{n} Q_j\right) \\ &= (p+\pi-c_{(i)})-(p+\pi-s)F\left(\sum_{j=1}^{(i-1)} K_j + Q_{(i)}\right) \\ &= 0\end{aligned}$$

which means that

$$Q_{(i)} = Y_{(i)} - \sum_{j=1}^{(i-1)} K_j$$

<u>Case 2:</u> $\lambda_{(i)} \neq 0$ *and* $Q_{(i)} = K_{(i)}$. *In this case, we can observe from (35) and (36) that* $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should satisfy the inequality*

$$\frac{\partial \Pi}{\partial Q_{(i)}} < 0.$$

However, this inequality is not true (see 43); hence, $Q_{(i)} = K_{(i)}$ *can not be a solution in this case. Therefore, the optimal quantity to order from supplier i such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *is* $Q^*_{(i)} = Y_{(i)} - \sum_{j=1}^{i-1} K_{(j)}$. *Proposition 8 is proved.* □

Proposition 5-8 indicates that under the third scenario, the retailer spreads its orders among less expensive suppliers. Once the capacities of all less expensive suppliers are exhausted, the retailer orders from supplier i whose role is to cover the remaining quantity required in such a way that the optimal ordering level $\sum_{j=1}^{n} Q_j$ is exactly equal to the threshold level Y_i.

4.1.2.4 Fourth scenario: $Y_{(i)} < \sum_{j=1}^{i-1} K_{(j)} < \sum_{j=1}^{i} K_{(j)}$ for $i = 1, ..., n$

Lemma 3 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1, ..., n$, *then* $Q_j^* = 0 \; \forall \; j > i$.

Proof 11 *The proof is exactly as presented in the proof of Proposition 5.* □

Lemma 4 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1, ..., n$, *then* $Q_j^* = K_{(j)} \; \forall \; j < i$.

Proof 12 *The proof is exactly as presented in the proof of Proposition 6.* □

Proposition 8 *Consider the model with n suppliers. If supplier i is given such that* $\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} < \sum_{j=1}^{i} K_{(j)}$ *for* $i = 1, ..., n$, *then* $Q_{(i)}^* = 0$.

Proof 13 *The proof is made by contradiction. Suppose that* $Q_i^* > 0$. *Hereafter, we will use* Q_i *to refer to* Q_i^* *in order to simplify the expressions. Now, it yields using (KKT) conditions (35) and (36) for the supplier i with a cost* $c_{(i)}$ *and capacity production* $K_{(i)}$*:*

$$\frac{\partial \Pi}{\partial Q_{(i)}} - \lambda_{(i)} = 0 \Rightarrow \frac{\partial \Pi}{\partial Q_{(i)}} = \lambda_{(i)}.$$

Using (KKT) condition (39), $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should be equal to zero or strictly positive where*

$$\begin{aligned} \frac{\partial \Pi}{\partial Q_{(i)}} &= (p + \pi - c_{(i)}) - (p + \pi - s) F\left(\sum_{j=1}^{n} Q_j\right) \\ &= (p + \pi - c_{(i)}) - (p + \pi - s) F\left(\sum_{j=1}^{(i-1)} K_j + Q_{(i)}\right) \end{aligned}$$

On one hand, if $\frac{\partial \Pi}{\partial Q_{(i)}} = 0$, *then*

$$F(\sum_{j=1}^{(i-1)} K_j + Q_{(i)}) = \frac{p + \pi - c_{(i)}}{p + \pi - s} \Rightarrow Q_{(i)} = Y_i - \sum_{j=1}^{(i-1)} K_j.$$

But, $Y_i < \sum_{j=1}^{(i-1)} K_j \Rightarrow Q_{(i)}^* < 0$ *which is impossible since we suppose at the beginning that* $Q_{(i)}^* > 0$.

On the other hand, if $\frac{\partial \Pi}{\partial Q_{(i)}} > 0$, *then*

$$F(\sum_{j=1}^{(i-1)} K_j + Q_{(i)}) < \frac{p + \pi - c_{(i)}}{p + \pi - s} \Rightarrow = Y_i > Q_{(i)} + \sum_{j=1}^{(i-1)} K_j.$$

But, we suppose that the optimal quantity of $Q_{(i)}$ *is positive and* $\sum_{j=1}^{(i-1)} K_j > 0 \Rightarrow Y_i > \sum_{j=1}^{(i-1)} K_j$. *which is impossible since we study the case where* $Y_i > \sum_{j=1}^{(i-1)} K_j$.

From (38), there are two cases to check:

<u>Case 1:</u> $\lambda_{(i)} = 0$ *and* $Q_{(i)} \neq K_{(i)}$ *then, condition (36) is reduced to*

$$Q_{(i)} \left(\frac{\partial \Pi}{\partial Q_{(i)}} \right) = 0 \tag{42}$$

Thus, we have two possibilities:

First possibility: Assume $Q_{(i)} = 0$. *In this case,* $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should be negative (according to condition (35)). But, we have*

$$\begin{aligned} \frac{\partial \Pi}{\partial Q_{(i)}} &= (p+\pi-c_{(i)}) - (p+\pi-s)F(\sum_{j=1}^{n} Q_j) \\ &= (p+\pi-c_{(i)}) - (p+\pi-s)F(\sum_{j=1}^{(i-1)} K_j) \text{ (thanks to Proposition 5 and Proposition 6)} \\ &> 0 \quad (\text{since } \sum_{j=1}^{i-1} K_{(j)} < Y_{(i)} \text{ and using the monotonicity of } F(.)) \end{aligned} \tag{43}$$

which violates condition (35).

Second possibility: Assume $Q_{(i)} \neq 0$. *In this case,* $\frac{\partial \Pi}{\partial Q_{(i)}}$ *should be equal zero. Hence,*

$$\begin{aligned} \frac{\partial \Pi}{\partial Q_{(i)}} &= (p+\pi-c_{(i)}) - (p+\pi-s)F\left(\sum_{j=1}^{n} Q_j\right) \\ &= (p+\pi-c_{(i)}) - (p+\pi-s)F\left(\sum_{j=1}^{(i-1)} K_j + Q_{(i)}\right) \\ &= 0 \end{aligned}$$

which means that

$$Q_{(i)} = Y_{(i)} - \sum_{j=1}^{(i-1)} K_j$$

4.2 Sustainability problem

In this section, we present the second stage represented by the sustainability problem where a supplier evaluation procedure is established in order to select appropriate suppliers who are the best in terms of preserving green and social aspects. As mentioned earlier, we use in the first step, fuzzy TOPSIS to rank potential suppliers on the basis of two sets of criteria: social and green. In the second step, top management (preferably) uses AHP to assign importance weights to each of the two sets of criteria based on the organization's strategy. Consequently, the potential supplier considered very poor in terms of green criteria and very good in terms of social criteria will not be ranked among the best alternatives if top management decides to give more importance to the set of green criteria. This approach is more general and provides more flexibility for decision makers in highlighting the importance of one set over the other [CIE2017cheaitouhamdan, 2015, 2017(b)].

Hereafter, we present the sustainable model which aims to maximize the green function denoted by W_{ST}. Moreover, we show that this function is jointly concave with respect to Q_i. In addition, we present the structure optimal policy for the sustainable model without details since it will follow the same structure already derived for the cost model.

4.2.1 Sustainability model and optimal ordering policy

The sustainability problem is to find the quantity which maximizes the green value W_{ST} where

$$\begin{aligned} W_{ST}(\mathbf{Q}) = \quad & \textstyle\sum_{i=1}^{n} \quad w_i^{GS} \times Q_i + w^c \int_0^{\sum_{i=1}^n Q_i} D f(D) dD + w^c \int_{\sum_{i=1}^n Q_i}^{\infty} (\sum_{i=1}^{n} Q_i) f(D) dD \\ - \quad & w^{GS} \int_0^{\sum_{i=1}^n Q_i} \left(\sum_{i=1}^{n} Q_i - D \right) f(D) dD \\ - \quad & \pi^c \int_{\sum_{i=1}^n Q_i}^{\infty} \left(D - \sum_{i=1}^{n} Q_i \right) f(D) dD. \end{aligned} \tag{44}$$

The retailer needs to solve the following optimization problem

$$\max W_{ST}(\mathbf{Q}) \tag{45}$$

$$\text{subject to } 0 \leq Q_i \leq K_i \quad \text{for all i.} \tag{46}$$

The partial derivatives of W_{ST} with respect to Q_i $(i = 1, ..., n)$ are

$$\frac{\partial W_{ST}(\mathbf{Q})}{\partial Q_i} = (w^c + \pi^c + w_i^{GS}) - (w^c + \pi^c + w^{GS}) F \left(\sum_{i=1}^{n} Q_i \right). \tag{47}$$

Setting the first partial derivatives (47) equal to zero, we obtain

$$F \left(\sum_{i=1}^{n} Q_i \right) = \frac{w^c + \pi^c + w_i^{GS}}{w^c + \pi^c + w^{GS}} \tag{48}$$

which implies that

$$\sum_{i=1}^{n} Q_i = F^{-1}\left(\frac{w^c+\pi^c+w_i^{GS}}{w^c+\pi^c+w^{GS}}\right). \tag{49}$$

Thus, we can define the following threshold level related to the sustainability problem

$$Y_i^{GS} = F^{-1}\left(\frac{w^c+\pi^c+w_i^{GS}}{w^c+\pi^c+w^{GS}}\right). \tag{50}$$

Similarly to the cost problem, we have suppliers are indexed such that

$$Y_{(n)}^{GS} < ... < Y_{(i)}^{GS} < ... < Y_{(1)}^{GS},$$

since $Y_i^{GS} = F^{-1}\left(\frac{w^c+\pi^c+w_i^{GS}}{w^c+\pi^c+w^{GS}}\right)$ and suppliers are indexed from the highest to lowest performance level in terms of green and social criteria such that

$$w_{(n)}^{GS} > ... > w_{(i)}^{GS} > ... > w_{(1)}^{GS}, \tag{51}$$

Next, we will show that the objective function $W_{ST}(Q)$ is jointly concave with respect to Q_i.

Proposition 9 *The expected objective function $W_{ST}(\boldsymbol{Q})$ defined in (44) is a jointly concave function with respect to Q_i, $i = 1,...,n$.*

Proof 14 *The Hessian matrix of $W_{ST}(\boldsymbol{Q})$ with respect to Q_i ($i = 1,...,n$) is the $n \times n$ matrix given by*

$$H_{W_{ST}}(\boldsymbol{Q}) = -(w^c + w^{GS} + \pi^c) f(\sum_{i=1}^{n} Q_i) J_n \tag{52}$$

where J_n is $(n \times n)$ matrix of ones.

For each vector

$$\boldsymbol{V} = (V_1,...,V_n) \subset \mathbb{R}^n,$$

it yields

$$\boldsymbol{V}^T \left(H_\Pi(\boldsymbol{Q})\right) \boldsymbol{V} = -(w^c + w^{GS} + \pi^c) f(\sum_{i=1}^{n} Q_i)(\sum_{i=1}^{n} V_i)^2 < 0.$$

Consequently, the matrix $H_{W_{ST}}(\boldsymbol{Q})$ is semi-definite negative; thus, the objective function $W_{ST}(\boldsymbol{Q})$ is jointly concave with respect to Q_i ($i = 1,...,n$), which completes the proof.

□ According to Proposition 9 and to the fact that the constraints are linear, the optimal solution can be found using (KKT) conditions. Hence, similar as in the cost problem where (KKT) conditions are applied ((10)-(14)), we can apply similarly these conditions the sustainability problem with the necessary modifications (which is by replacing in the conditions the function Π by W_{ST}).

Concerning the optimal ordering strategy for the sustainability problem, it can be seen from (50) that the threshold level Y_i^{GS} related to the sustainability problem follows the same structure of the threshold (6) related to the cost model; in fact, by a simple comparison, we can notice that p is replaced here by w^c, π is replaced here by π^c and c_i is replaced here by w_i^{GS}. Thus, based also on what we just said previously on the optimality conditions of (KKT), we expect naturally that the structure of the optimal policy for the sustainability model follows the same structure already detailed for the cost model. Therefore, we can imitate the structure optimal ordering policy done for the cost model with the necessary modifications. Details are omitted for brevity. However, to make it more clear, we summarize in Table 2 the retailer's optimal ordering policy for the sustainability model:

Table 2: Retailer's optimal ordering policy for the sustainability model

Case	Optimal solution
$Y_{(i+1)}^{GS} < \sum_{j=1}^{i} K_{(j)} < Y_{(i)}^{GS}$	$Q_{(j)}^* = K_{(j)}\ \forall\ j \leq i$ and $Q_{(j)}^* = 0\ \forall\ j > i$
$\sum_{j=1}^{i-1} K_{(j)} \leq \sum_{j=1}^{i} K_{(j)} < Y_{(i+1)}^{GS} \leq Y_{(i)}^{GS}$	$Q_{(j)}^* = K_{(j)}\ \forall\ j \leq i$ and $Q_{(j)}^* > 0\ \forall\ j > i$
$\sum_{j=1}^{i-1} K_{(j)} < Y_{(i)}^{GS} < \sum_{j=1}^{i} K_{(j)}$	$Q_j^* = K_{(j)}\ \forall\ j < i$, $Q_{(i)}^* = Y_{(i)}^{GS} - \sum_{j=1}^{i-1} K_{(j)}$ and $Q_j^* = 0\ \forall\ j > i$
$Y_{(i)}^{GS} < \sum_{j=1}^{i-1} K_{(j)} < \sum_{j=1}^{i} K_{(j)}$	$Q_j^* = K_{(j)}\ \forall\ j < i$, $Q_j^* = 0\ \forall\ j \geq i$.

4.3 Bi-objective integer linear programming model

In this section, we present the third and final stage where the outputs of the first and second stages serve as inputs for a single-product bi-objective optimization model, which maximizes the expected profit (and thus minimizes the suppliers' cost) and maximizes the preference weights of the selected suppliers. The model is solved using the weighted comprehensive criterion method (See CIE (cheaitou page 2)—>Dehghani, Esmaeilian, Tavakkoli- Moghaddam, 2013).

4.3.1 Bi-objective model and solution approach

The bi-objective optimization model consists of two objective functions. The first objective function aims to maximize the expected profit Π defined in Eq. (2). The second objective function aims to maximize the green function W_{ST} defined in Eq. (44). Thus, the bi-objective model is defined as follows:

$$\max \Pi \tag{53}$$

$$\max W_{ST} \tag{54}$$

$$\text{subject to} \quad 0 \le Q_i \le K_i \quad \forall i = 1, ..., n \tag{55}$$

In order to solve the bi-objective model described previously, we can choose between many different scalarization techniques such that the weighted sum method [reference], the ε-constraint method [reference], the comprehensive criterion method [reference], or the weighted comprehensive criterion method denoted by (WCCM) [ref,ref]. In this paper, we adopt the WCCM because of the simplicity of its implementation and its efficiency, in terms of the number of Pareto solutions it can provide (Kamali et al., 2011; Marler, Arora, 2004). We will not detail here all the steps of the WCCM (we invite the reader to see (HamdanCheaitou) for more details) and we will only present the final normalized objective function which will be minimized subject to the same constraints of the two single objective functions problems. Finally, it infer

$$\begin{aligned} \min Z &= 100 \times \left(\frac{\Pi^* - \Pi}{\Pi^*} \omega_1 + \frac{W_{ST}^* - W_{ST}}{W_{ST}^*} \omega_2 \right) \\ &= 100 \times \left(-\frac{\omega_1}{\Pi^*} \Pi - \frac{\omega_2}{W_{ST}^*} W_{ST} + \omega_1 + \omega_2 \right), \end{aligned} \tag{56}$$

$$\text{subject to} \quad 0 \le Q_i \le K_i \quad \forall i = 1, ..., n \tag{57}$$

where ω_1 and ω_2 are the relative weights that the decision maker can set with $\omega_1 + \omega_2 = 1$. In addition, Π and W_{ST} can be substituted in (56) by their respective expressions (see Eqs. (2) and (44)).

It is very interesting to note that for a given value of ω_1 and ω_2, the function Z is a linear combination of Π and W_{ST}. This implies that

- The function Z is jointly concave with respect to Q_i since it is the sum of two functions jointly concave (thanks to Propositions 1 and 9).
- The partial derivative of the function Z is a direct consequence of Eqs. (5) and (47) and this is using the linearity of differentiation. Hence, for a given value of ω_1 and ω_2, the partial derivative of Z with respect to Q_i can be written as

$$\begin{aligned} \frac{\partial Z}{\partial Q_i} &= -\frac{\omega_1}{\Pi^*} \frac{\partial \Pi}{\partial Q_i} - \frac{\omega_2}{W_{ST}^*} \frac{\partial W_{ST}}{\partial Q_i} \\ &= -\frac{\omega_1}{\Pi^*} \left[(p + \pi - c_i) - (p + \pi - s) F\left(\sum_{i=1}^{n} Q_i \right) \right] \\ &- \frac{\omega_2}{W_{ST}^*} \left[(w^c + \pi^c + w_i^{GS}) - (w^c + \pi^c + w^{GS}) F\left(\sum_{i=1}^{n} Q_i \right) \right] \end{aligned} \tag{58}$$

Rearranging (58), it yields

$$
\begin{aligned}
\frac{\partial Z}{\partial Q_i} &= \left[(-\frac{\omega_1}{\Pi^*})(p+\pi-c_i)+(-\frac{\omega_2}{W^*_{ST}})(w^c+\pi^c+w_i^{GS})\right] \\
&- \left[(-\frac{\omega_1}{\Pi^*})(p+\pi-s)+(-\frac{\omega_2}{W^*_{ST}})(w^c+\pi^c+w^{GS})\right] F\left(\sum_{i=1}^{n} Q_i\right)
\end{aligned} \quad (59)
$$

Setting (59) equal to zero, we derive

$$
F\left(\sum_{i=1}^{n} Q_i\right) = \frac{\frac{\omega_1}{\Pi^*}(p+\pi-c_i)+\frac{\omega_2}{W^*_{ST}}(w^c+\pi^c+w_i^{GS})}{\frac{\omega_1}{\Pi^*}(p+\pi-s)+\frac{\omega_2}{W^*_{ST}}(w^c+\pi^c+w^{GS})} \quad (60)
$$

From (60), we can define the following threshold related to the bi-objective problem

$$
Y_i^{WCCM} = F^{-1}\left(\frac{\frac{\omega_1}{\Pi^*}(p+\pi-c_i)+\frac{\omega_2}{W^*_{ST}}(w^c+\pi^c+w_i^{GS})}{\frac{\omega_1}{\Pi^*}(p+\pi-s)+\frac{\omega_2}{W^*_{ST}}(w^c+\pi^c+w^{GS})}\right) \quad (61)
$$

It is worth noting that Equation (61) has the same structure as the single objective sub-problems thresholds, which allows to use the same optimal policy structure found for the single objective problems for this bi-objective model. Moreover, it is also worth noting that the sorted Y_i^{WCCM} from the largest value to the smallest value corresponds to the best combination of cost/profit and sustainability value.

5 Numerical example

In this example we provide a simple numerical example to show the way the model works. This basic numerical example considers five suppliers with the following parameters:

Table 3: Input data of the numerical example

Supplier i	K_i	c_i	w_i
1	250	29	0.06
2	200	22	0.04
3	200	16	0.1
4	900	32	0.6
5	1200	20	0.2

Moreover, the other parameters are as follows: $p = 75$, $s = 10$, $\pi = 20$, $w^{GS} = 0.5$, $\pi^c = 0.3$, and $w^c = 0.2$.

For the demand, we assumed a normal distribution with a mean $\mu = 1000$ and a standard deviation of $\sigma = 300$.

The single objective optimal solution are as follows: 50766.2 and 364.352 for the optimal cost and sustainability objective functions respectively. Moreover, the corresponding thresholds and optimal quantities are as follows:

Table 4: Optimal solution of the cost and sustainability sub-problems

Supplier i	Y_i	Y_i^{GS}	Q_i of the cost sub-problem	Q_i of the sustainability sub-problem
1	1228.1	1022.58	0	0
2	1322.51	1015.05	0	0
3	1441.43	1037.7	200	0
4	1194.09	1252.49	0	900
5	1356.05	1076.	1156.05	176.004

For the bi-objective problem, we varied the weights ω_1 and ω_2 from 0.2 to 0.9 and from 0.8 to 0.1 respectively with a step of 0.1. The obtained Pareto optimal solutions and their corresponding variation are shown in Table 5.

Table 5: Pareto optimal solutions

ω_1	ω_2	Z $(\%)$	Optimal ordered quantities (Q_i)
0.2	0.8	5.61064%	0, 0, 0, 900, 205.426
0.3	0.7	8.29667%	0, 0, 0, 900, 222.529
0.4	0.6	10.8859%	0, 0, 0, 900, 241.681
0.5	0.5	13.361%	0, 0, 0, 900, 263.378
0.6	0.4	15.6996%	0, 0, 0, 900, 288.325
0.7	0.3	17.521%	0, 0, 200, 0, 1017.57
0.8	0.2	11.9848%	0, 0, 200, 0, 1052.78
0.9	0.1	6.17806%	0, 0, 200, 0, 1096.92

This numerical example shows the flexibility that this framework can offer to the purchasing departments' decision makers in choosing the best suppliers and their corresponding quantities based on the importance that they give to the objective functions.

6 Conclusion

Including sustainability aspects in the decision making processes of most of the businesses has become a necessity. Logistics as a major contributor of green house emissions also should contribute in this regard and as part of logistics inventory systems also are expected to contribute in reducing the negative impacts of business on the environment and the society. This paper contributes in this direction by proposing a bi-objective single-product multiple-supplier newsvendor model in which profit and the sustainability value of the purchased items are optimized. First, each sub-problem is solved and its optimal policy is characterized. The optimal solutions and then used to characterize the optimal policy of the bi-objective model. The obtained optimal policies are based on a threshold structure and can be used very simply. A numerical example is solved to show the effectiveness of the proposed approach. The results show the flexibility that this approach offers to the decision makers in selecting the best suppliers and their corresponding quantities.

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